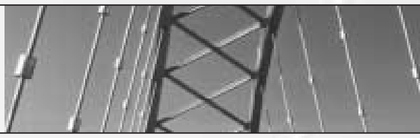
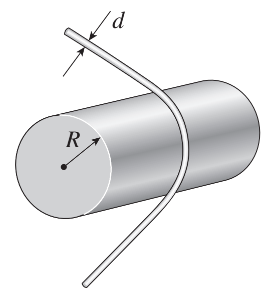


5

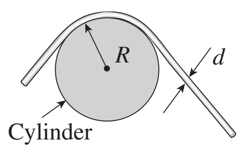
Stresses in Beams (Basic Topics)

Longitudinal Strains in Beams

Problem 5.4-1 Determine the maximum normal strain ϵ_{max} produced in a steel wire of diameter $d = 1/16$ in. when it is bent around a cylindrical drum of radius $R = 24$ in. (see figure).



Solution 5.4-1 **Steel wire**



$R = 24 \text{ in.} \qquad d = \frac{1}{16} \text{ in.}$

From Eq. (5-4):

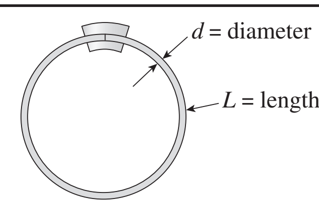
$$\varepsilon_{max} = \frac{y}{\rho}$$

$$= \frac{d/2}{R + d/2} = \frac{d}{2R + d}$$

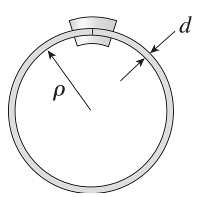
Substitute numerical values:

$$\varepsilon_{max} = \frac{1/16 \text{ in.}}{2(24 \text{ in.}) + 1/16 \text{ in.}} = 1300 \times 10^{-6} \quad \longleftarrow$$

Problem 5.4-2 A copper wire having diameter $d = 3$ mm is bent into a circle and held with the ends just touching (see figure). If the maximum permissible strain in the copper is $\epsilon_{max} = 0.0024$, what is the shortest length L of wire that can be used?



Solution 5.4-2 **Copper wire**



$d = 3 \text{ mm} \qquad \varepsilon_{max} = 0.0024$

$L = 2\pi\rho \qquad \rho = \frac{L}{2\pi}$

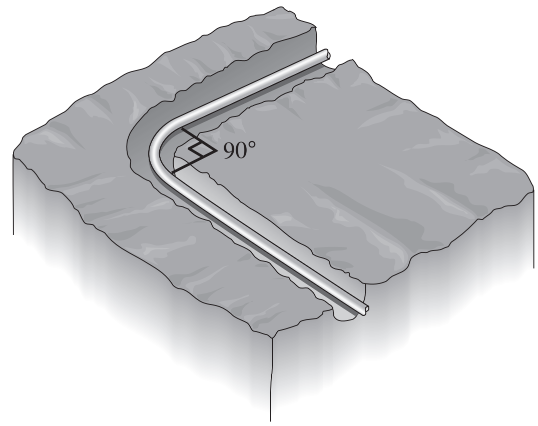
From Eq. (5-4):

$$\varepsilon_{max} = \frac{y}{\rho} = \frac{d/2}{L/2\pi} = \frac{\pi d}{L}$$

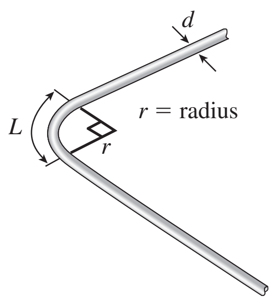
$$L_{min} = \frac{\pi d}{\varepsilon_{max}} = \frac{\pi \text{ (3 mm)}}{0.0024} = 3.93 \text{ m} \quad \longleftarrow$$

Problem 5.4-3 A 4.5 in. outside diameter polyethylene pipe designed to carry chemical wastes is placed in a trench and bent around a quarter-circular 90° bend (see figure). The bent section of the pipe is 46 ft long.

Determine the maximum compressive strain ϵ_{max} in the pipe.



Solution 5.4-3 **Polyethylene pipe**



$L =$ length of 90º bend

$L = 46\text{ ft} = 552\text{ in.}$

$d = 4.5\text{ in.}$

$$L = \frac{2\pi r}{4} = \frac{\pi r}{2}$$

Angle equals 90º or $\pi/2$ radians,

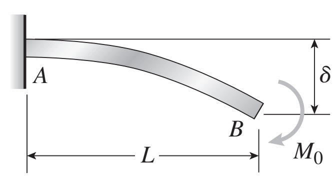
$r = \rho =$ radius of curvature

$$\rho = \frac{L}{\pi/2} = \frac{2L}{\pi} \qquad \varepsilon_{max} = \frac{y}{\rho} = \frac{d/2}{2L/\pi}$$

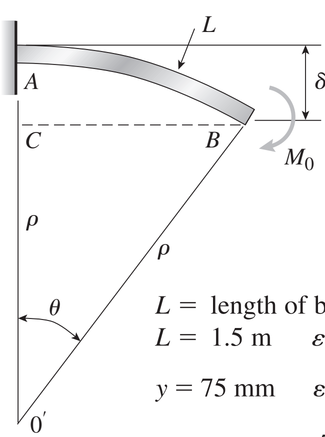
$$\varepsilon_{max} = \frac{\pi d}{4L} = \frac{\pi}{4}\left(\frac{4.5\text{ in.}}{552\text{ in.}}\right) = 6400 \times 10^{-6} \quad \longleftarrow$$

Problem 5.4-4 A cantilever beam AB is loaded by a couple M_0 at its free end (see figure). The length of the beam is $L = 1.5$ m and the longitudinal normal strain at the top surface is 0.001. The distance from the top surface of the beam to the neutral surface is 75 mm.

Calculate the radius of curvature ρ, the curvature κ, and the vertical deflection δ at the end of the beam.



Solution 5.4-4 **Cantilever beam**



$L =$ length of beam

$L = 1.5\text{ m} \qquad \varepsilon_{max} = 0.001$

$$y = 75\text{ mm} \qquad \varepsilon_{max} = \frac{y}{\rho}$$

$$\therefore \rho = \frac{y}{\varepsilon_{max}} = \frac{75\text{ mm}}{0.001} = 75\text{ m} \quad \longleftarrow$$

$$\kappa = \frac{1}{\rho} = 0.01333\text{ m}^{-1} \quad \longleftarrow$$

Assume that the deflection curve is nearly flat. Then the distance BC is the same as the length L of the beam.

$$\therefore \sin\theta = \frac{L}{\rho} = \frac{1.5\text{ m}}{75\text{ m}} = 0.02$$

$\theta = \arcsin 0.02 = 0.02$ rad

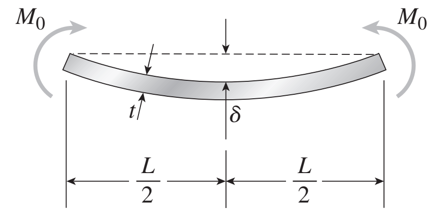
$\delta = \rho(1 - \cos\theta) = (75\text{ m})(1 - \cos(0.02\text{ rad}))$

$= 15.0\text{ mm} \quad \longleftarrow$

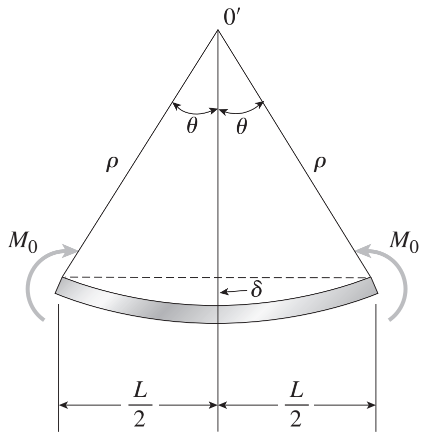
NOTE: $\dfrac{L}{\delta} = 100$, which confirms that the deflection curve is nearly flat.

Problem 5.4-5 A thin strip of steel of length $L = 20$ in. and thickness $t = 0.2$ in. is bent by couples M_0 (see figure). The deflection δ at the midpoint of the strip (measured from a line joining its end points) is found to be 0.25 in.

Determine the longitudinal normal strain ϵ at the top surface of the strip.



Solution 5.4-5 Thin strip of steel



$L = 20$ in. $t = 0.2$ in.
$\delta = 0.25$ in.

The deflection curve is very flat (note that $L/\delta = 80$) and therefore θ is a very small angle.

$$\sin\theta = \frac{L/2}{\rho}$$

For small angles, $\theta = \sin\theta = \dfrac{L/2}{\rho}$ (θ is in radians)

$$\delta = \rho - \rho\cos\theta = \rho(1 - \cos\theta)$$
$$= \rho\left(1 - \cos\frac{L}{2\rho}\right)$$

Substitute numerical values ($\rho =$ inches):

$$0.25 = \rho\left(1 - \cos\frac{10}{\rho}\right)$$

Solve numerically: $\rho = 200.0$ in.

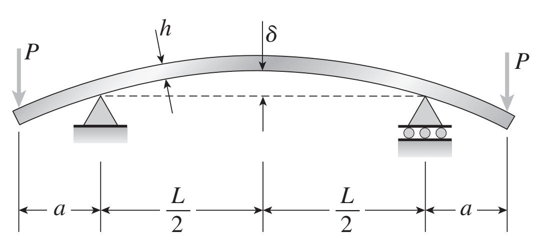
NORMAL STRAIN

$$\varepsilon = \frac{y}{\rho} = \frac{t/2}{\rho} = \frac{0.1\text{ in.}}{200\text{ in.}} = 500 \times 10^{-6} \quad \longleftarrow$$

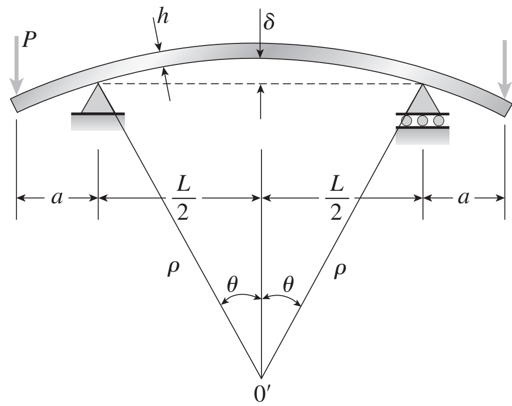
(Shortening at the top surface)

Problem 5.4-6 A bar of rectangular cross section is loaded and supported as shown in the figure. The distance between supports is $L = 1.2$ m and the height of the bar is $h = 100$ mm. The deflection δ at the midpoint is measured as 3.6 mm.

What is the maximum normal strain ϵ at the top and bottom of the bar?



Solution 5.4-6 **Bar of rectangular cross section**



$L = 1.2$ m $h = 100$ mm $\delta = 3.6$ mm

Note that the deflection curve is nearly flat ($L/\delta = 333$) and θ is a very small angle.

$$\sin\theta = \frac{L/2}{\rho}$$

$$\theta = \frac{L/2}{\rho} \text{ (radians)}$$

$$\delta = \rho\,(1 - \cos\theta) = \rho\left(1 - \cos\frac{L}{2\rho}\right)$$

Substitute numerical values ($\rho =$ meters):

$$0.0036 = \rho\left(1 - \cos\frac{0.6}{\rho}\right)$$

Solve numerically: $\rho = 50.00$ m

NORMAL STRAIN

$$\varepsilon = \frac{y}{\rho} = \frac{h/2}{\rho} = \frac{50 \text{ mm}}{50{,}000 \text{ mm}} = 1000 \times 10^{-6} \quad \longleftarrow$$

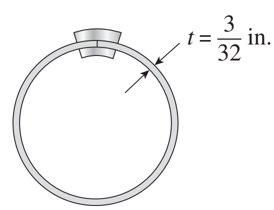
(Elongation on top; shortening on bottom)

Normal Stresses in Beams

Problem 5.5-1 A thin strip of hard copper ($E = 16{,}400$ ksi) having length $L = 80$ in. and thickness $t = 3/32$ in. is bent into a circle and held with the ends just touching (see figure).

(a) Calculate the maximum bending stress σ_{max} in the strip.

(b) Does the stress increase or decrease if the thickness of the strip is increased?



Solution 5.5-1 **Copper strip bent into a circle**

$E = 16{,}400$ ksi $L = 80$ in. $t = 3/32$ in.

(a) MAXIMUM BENDING STRESS

$$L = 2\pi r = 2\pi\rho \qquad \rho = \frac{L}{2\pi}$$

From Eq. (5-7): $\sigma = \dfrac{Ey}{\rho} = \dfrac{2\pi E y}{L}$

$$\sigma_{max} = \frac{2\pi E(t/2)}{L} = \frac{\pi E t}{L}$$

Substitute numerical values:

$$\sigma_{max} = \frac{\pi\,(16{,}400 \text{ ksi})(3/32 \text{ in.})}{80 \text{ in.}} = 60.4 \text{ ksi} \quad \longleftarrow$$

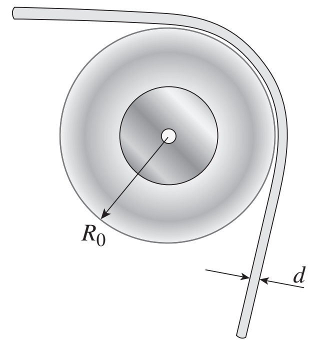
(b) CHANGE IN STRESS

If the thickness t is increased, the stress σ_{max} increases. ⟵

Problem 5.5-2 A steel wire ($E = 200$ GPa) of diameter $d = 1.0$ mm is bent around a pulley of radius $R_0 = 400$ mm (see figure).

(a) What is the maximum stress σ_{max} in the wire?

(b) Does the stress increase or decrease if the radius of the pulley is increased?



Solution 5.5-2 **Steel wire bent around a pulley**

$E = 200$ GPa $\quad d = 1.0$ mm $\quad R_0 = 400$ mm

(a) MAXIMUM STRESS IN THE WIRE

$$\rho = R_0 + \frac{d}{2} = 400 \text{ mm} + 0.5 \text{ mm} = 400.5 \text{ mm}$$

$$y = \frac{d}{2} = 0.5 \text{ mm}$$

From Eq. (5-7):

$$\sigma_{max} = \frac{Ey}{\rho} = \frac{(200 \text{ GPa})\ (0.5 \text{ mm})}{400.5 \text{ mm}} = 250 \text{ MPa} \quad \longleftarrow$$

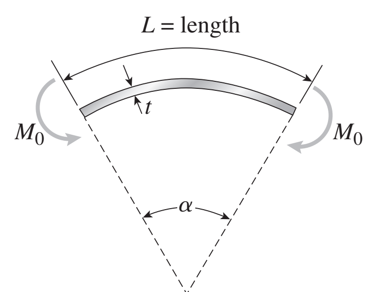
(b) CHANGE IN STRESS

If the radius is increased, the stress σ_{max} decreases. ⟵

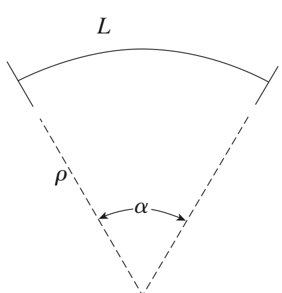
Problem 5.5-3 A thin, high-strength steel rule ($E = 30 \times 10^6$ psi) having thickness $t = 0.15$ in. and length $L = 40$ in. is bent by couples M_0 into a circular arc subtending a central angle $\alpha = 45°$ (see figure).

(a) What is the maximum bending stress σ_{max} in the rule?

(b) Does the stress increase or decrease if the central angle is increased?



Solution 5.5-3 **Thin steel rule bent into an arc**



$E = 30 \times 10^6$ psi

$t = 0.15$ in.

$L = 40$ in.

$\alpha = 45° = 0.78540$ rad

(a) MAXIMUM BENDING STRESS

$$L = \rho\alpha \qquad \rho = \frac{L}{\alpha} \qquad \alpha = \text{radians}$$

$$\sigma_{max} = \frac{Ey}{\rho} = \frac{E(t/2)}{L/\alpha} = \frac{Et\alpha}{2L}$$

Substitute numerical values:

$$\sigma_{max} = \frac{(30 \times 10^6 \text{ psi})\ (0.15 \text{ in.})\ (0.78540 \text{ rad})}{2\ (40 \text{ in.})}$$

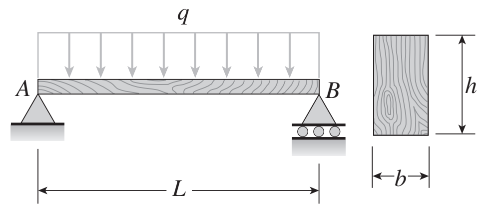
$$= 44{,}200 \text{ psi} = 44.2 \text{ ksi} \quad \longleftarrow$$

(b) CHANGE IN STRESS

If the angle α is increased, the stress σ_{max} increases. ⟵

Problem 5.5-4 A simply supported wood beam AB with span length $L = 3.5$ m carries a uniform load of intensity $q = 6.4$ kN/m (see figure).

Calculate the maximum bending stress σ_{max} due to the load q if the beam has a rectangular cross section with width $b = 140$ mm and height $h = 240$ mm.



Solution 5.5-4 **Simple beam with uniform load**

$L = 3.5$ m $\quad q = 6.4$ kN/m
$b = 140$ mm $\quad h = 240$ mm

$$M_{max} = \frac{qL^2}{8} \qquad S = \frac{bh^2}{6}$$

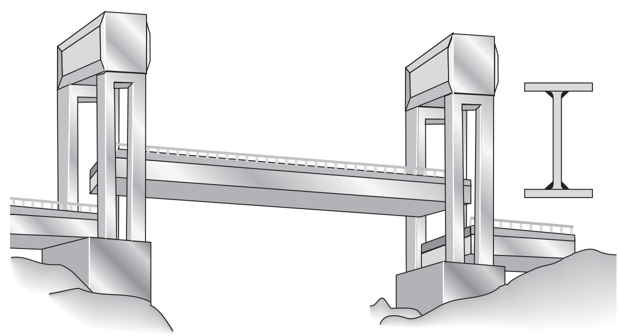
$$\sigma_{max} = \frac{M_{max}}{S} = \frac{3qL^2}{4bh^2}$$

Substitute numerical values:

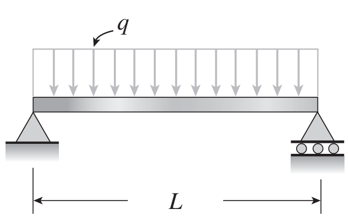
$$\sigma_{max} = \frac{3(6.4\text{ kN/m})(3.5\text{ m})^2}{4(140\text{ mm})(240\text{ mm})^2} = 7.29\text{ MPa} \quad \leftarrow$$

Problem 5.5-5 Each girder of the lift bridge (see figure) is 180 ft long and simply supported at the ends. The design load for each girder is a uniform load of intensity 1.6 k/ft. The girders are fabricated by welding three steel plates so as to form an I-shaped cross section (see figure) having section modulus $S = 3600$ in^3.

What is the maximum bending stress σ_{max} in a girder due to the uniform load?



Solution 5.5-5 **Bridge girder**



$L = 180$ ft $\quad q = 1.6$ k/ft

$S = 3600$ in.^{3}

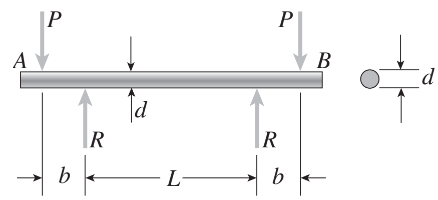
$$M_{max} = \frac{qL^2}{8}$$

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{qL^2}{8S}$$

$$\sigma_{max} = \frac{(1.6\text{ k/ft})(180\text{ ft})^2(12\text{ in./ft})}{8(3600\text{ in.}^3)} = 21.6\text{ ksi} \quad \leftarrow$$

Problem 5.5-6 A freight-car axle AB is loaded approximately as shown in the figure, with the forces P representing the car loads (transmitted to the axle through the axle boxes) and the forces R representing the rail loads (transmitted to the axle through the wheels). The diameter of the axle is $d = 80$ mm, the distance between centers of the rails is L, and the distance between the forces P and R is $b = 200$ mm.

Calculate the maximum bending stress σ_{max} in the axle if $P = 47$ kN.



Solution 5.5-6 Freight-car axle

Diameter $d = 80$ mm
Distance $b = 200$ mm
Load $P = 47$ kN

$$M_{max} = Pb \quad S = \frac{\pi d^3}{32}$$

MAXIMUM BENDING STRESS

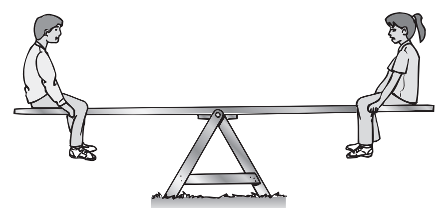
$$\sigma_{max} = \frac{M_{max}}{S} = \frac{32Pb}{\pi d^3}$$

Substitute numerical values:

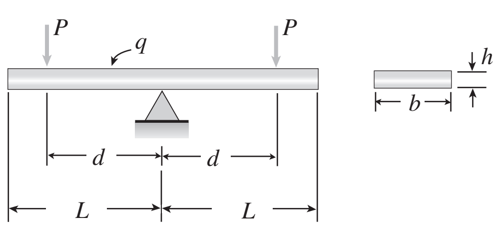
$$\sigma_{max} = \frac{32(47 \text{ kN})(200 \text{ mm})}{\pi(80 \text{ mm})^3} = 187 \text{ MPa} \quad \longleftarrow$$

Problem 5.5-7 A seesaw weighing 3 lb/ft of length is occupied by two children, each weighing 90 lb (see figure). The center of gravity of each child is 8 ft from the fulcrum. The board is 19 ft long, 8 in. wide, and 1.5 in. thick.

What is the maximum bending stress in the board?



Solution 5.5-7 Seesaw



$b = 8$ in. $\quad h = 1.5$ in.
$q = 3$ lb/ft $\quad P = 90$ lb $\quad d = 8.0$ ft $\quad L = 9.5$ ft

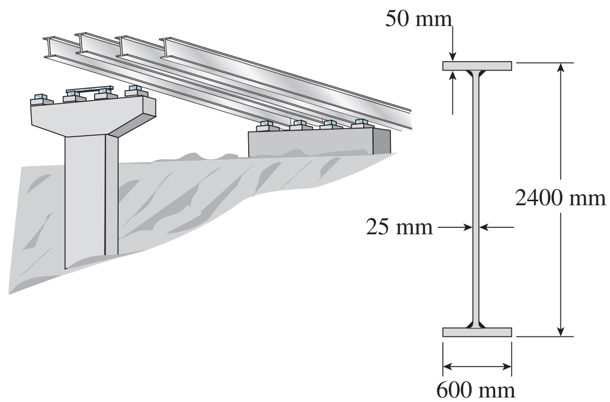
$$M_{max} = Pd + \frac{qL^2}{2} = 720 \text{ lb-ft} + 135.4 \text{ lb-ft}$$
$$= 855.4 \text{ lb-ft} = 10{,}264 \text{ lb-in.}$$

$$S = \frac{bh^2}{6} = 3.0 \text{ in}^3.$$

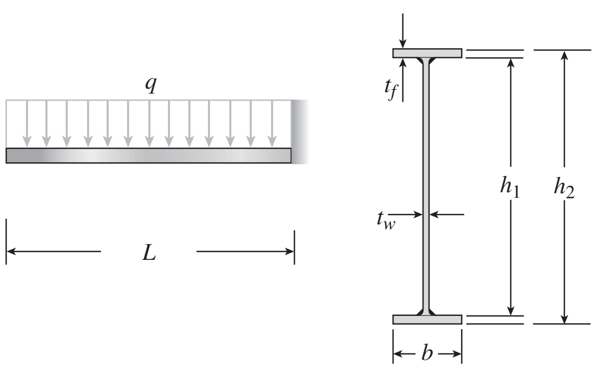
$$\sigma_{max} = \frac{M}{S} = \frac{10{,}264 \text{ lb-in.}}{3.0 \text{ in.}^3} = 3420 \text{ psi} \quad \longleftarrow$$

Problem 5.5-8 During construction of a highway bridge, the main girders are cantilevered outward from one pier toward the next (see figure). Each girder has a cantilever length of 46 m and an I-shaped cross section with dimensions as shown in the figure. The load on each girder (during construction) is assumed to be 11.0 kN/m, which includes the weight of the girder.

Determine the maximum bending stress in a girder due to this load.



Solution 5.5-8 **Bridge girder**



$L = 46$ m
$q = 11.0$ kN/m
$b = 600$ mm $h = 2400$ mm
$t_f = 50$ mm $t_w = 25$ mm
$h_1 = h - 2t_f = 2300$ mm
$b_1 = b - t_w = 575$ mm

$$M_{max} = \frac{qL^2}{2} = \frac{1}{2}(11.0 \text{ kN/m})(46 \text{ m})^2 = 11{,}638 \text{ kN} \cdot \text{m}$$

$$\sigma_{max} = \frac{M_{max}\,c}{I} \qquad c = \frac{h}{2} = 1200 \text{ mm}$$

$$I = \frac{bh^3}{12} - \frac{b_1 h_1^3}{12}$$

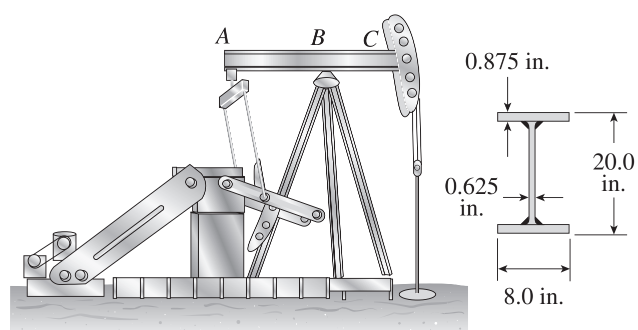
$$= \frac{1}{12}(0.6 \text{ m})(2.4 \text{ m})^3 - \frac{1}{12}(0.575 \text{ m})(2.3 \text{ m})^3$$

$$= 0.6912 \text{ m}^4 - 0.5830 \text{ m}^4 = 0.1082 \text{ m}^4$$

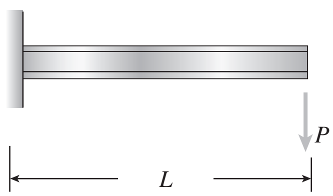
$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{(11{,}638 \text{ kN} \cdot \text{m})(1.2 \text{ m})}{0.1082 \text{ m}^4}$$

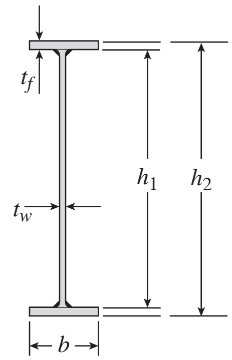
$$= 129 \text{ MPa} \longleftarrow$$

Problem 5.5-9 The horizontal beam *ABC* of an oil-well pump has the cross section shown in the figure. If the vertical pumping force acting at end *C* is 8.8 k, and if the distance from the line of action of that force to point *B* is 14 ft, what is the maximum bending stress in the beam due to the pumping force?



Solution 5.5-9 **Beam in an oil-well pump**





$L = 14$ ft
$P = 8.8$ k
$b = 8.0$ in. $\quad h = 20.0$ in.
$t_f = 0.875$ in. $\quad t_w = 0.625$ in.
$h_1 = h - 2t_f = 18.25$ in.
$b_1 = b - t_w = 7.375$ in.

$$M_{max} = PL = (8.8\text{ k})(14\text{ ft}) = 123{,}200\text{ lb-ft} = 1{,}478{,}400\text{ lb-in.}$$

$$\sigma_{max} = \frac{M_{max}\,c}{I} \qquad c = \frac{h}{2} = 10.0\text{ in.}$$

$$I = \frac{bh^3}{12} - \frac{b_1h_1^3}{12}$$

$$= \frac{1}{12}(8.0\text{ in.})(20.0\text{ in.})^3 - \frac{1}{12}(7.375\text{ in.})(18.25\text{ in.})^3$$

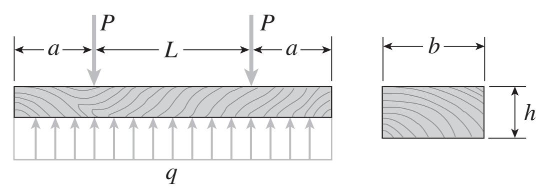
$$= 5{,}333.3\text{ in.}^4 - 3{,}735.7\text{ in.}^4 = 1{,}597.7\text{ in.}^4$$

$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{(1.4784 \times 10^6\text{ lb-in.})(10.0\text{ in.})}{1{,}597.7\text{ in.}^4}$$

$$= 9250\text{ psi} = 9.25\text{ ksi} \longleftarrow$$

Problem 5.5-10 A railroad tie (or *sleeper*) is subjected to two rail loads, each of magnitude $P = 175$ kN, acting as shown in the figure. The reaction q of the ballast is assumed to be uniformly distributed over the length of the tie, which has cross-sectional dimensions $b = 300$ mm and $h = 250$ mm.

Calculate the maximum bending stress σ_{max} in the tie due to the loads P, assuming the distance $L = 1500$ mm and the overhang length $a = 500$ mm.

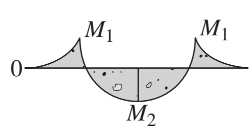


Solution 5.5-10 **Railroad tie (or sleeper)**

DATA $P = 175$ kN $\quad b = 300$ mm $\quad h = 250$ mm
$L = 1500$ mm $\quad a = 500$ mm

$$q = \frac{2P}{L + 2a} \qquad S = \frac{bh^2}{6} = 3.125 \times 10^{-3}\text{ m}^3$$

BENDING-MOMENT DIAGRAM



$$M_1 = \frac{qa^2}{2} = \frac{Pa^2}{L + 2a}$$

$$M_2 = \frac{q}{2}\left(\frac{L}{2} + a\right)^2 - \frac{PL}{2}$$

$$= \frac{P}{L + 2a}\left(\frac{L}{2} + a\right)^2 - \frac{PL}{2}$$

$$= \frac{P}{4}(2a - L)$$

Substitute numerical values:

$M_1 = 17{,}500$ N · m $\quad M_2 = -21{,}875$ N · m
$M_{max} = 21{,}875$ N · m

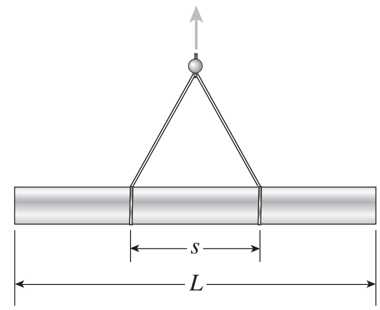
MAXIMUM BENDING STRESS

$$\sigma_{max} = \frac{M_{max}}{5} = \frac{21{,}875\text{ N}\cdot\text{m}}{3.125 \times 10^{-3}\text{ m}^3} = 7.0\text{ MPa} \longleftarrow$$

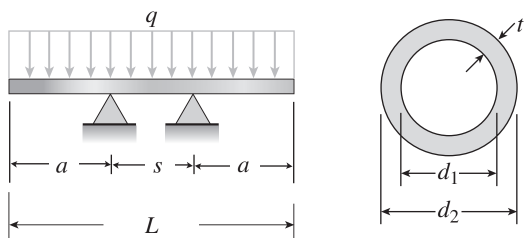
(Tension on top; compression on bottom)

Problem 5.5-11 A fiberglass pipe is lifted by a sling, as shown in the figure. The outer diameter of the pipe is 6.0 in., its thickness is 0.25 in., and its weight density is 0.053 lb/in.^{3} The length of the pipe is $L = 36$ ft and the distance between lifting points is $s = 11$ ft.

Determine the maximum bending stress in the pipe due to its own weight.



Solution 5.5-11 **Pipe lifted by a sling**



$L = 36 \text{ ft} = 432 \text{ in.}$ $d_2 = 6.0 \text{ in.}$ $t = 0.25 \text{ in.}$

$s = 11 \text{ ft} = 132 \text{ in.}$ $d_1 = d_2 - 2t = 5.5 \text{ in.}$

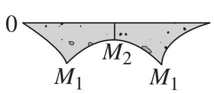
$\gamma = 0.053 \text{ lb/in.}^3$ $A = \frac{\pi}{4}(d_2^2 - d_1^2) = 4.5160 \text{ in.}^2$

$a = (L - s)/2 = 150 \text{ in.}$

$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 18.699 \text{ in.}^4$

$q = \gamma A = (0.053 \text{ lb/in.}^3)(4.5160 \text{ in.}^2) = 0.23935 \text{ lb/in.}$

BENDING-MOMENT DIAGRAM



$$M_1 = -\frac{qa^2}{2} = -2{,}692.7 \text{ lb-in.}$$

$$M_2 = -\frac{qL}{4}\left(\frac{L}{2} - s\right) = -2{,}171.4 \text{ lb-in.}$$

$M_{max} = 2{,}692.7 \text{ lb-in.}$

MAXIMUM BENDING STRESS

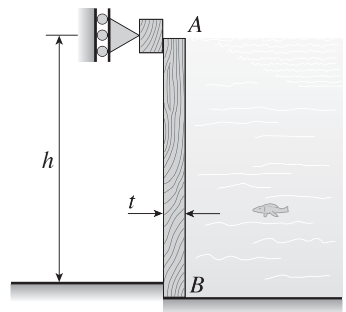
$$\sigma_{max} = \frac{M_{max}\, c}{I} \qquad c = \frac{d_2}{2} = 3.0 \text{ in.}$$

$$\sigma_{max} = \frac{(2{,}692.7 \text{ lb-in.})(3.0 \text{ in.})}{18.699 \text{ in.}^4} = 432 \text{ psi} \quad \longleftarrow$$

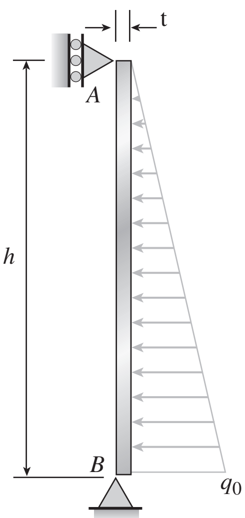
(Tension on top)

Problem 5.5-12 A small dam of height $h = 2.0$ m is constructed of vertical wood beams AB of thickness $t = 120$ mm, as shown in the figure. Consider the beams to be simply supported at the top and bottom.

Determine the maximum bending stress σ_{max} in the beams, assuming that the weight density of water is $\gamma = 9.81$ kN/m^3.



Solution 5.5-12 **Vertical wood beam**



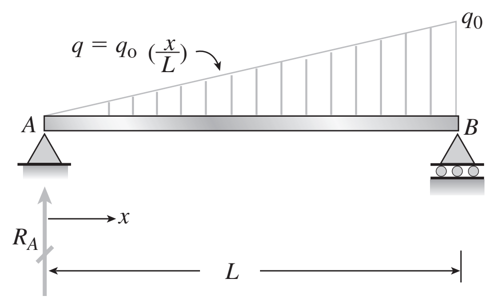
$h = 2.0$ m
$t = 120$ mm
$\gamma = 9.81$ kN/m^3 (water)

Let b = width of beam perpendicular to the plane of the figure

Let q_0 = maximum intensity of distributed load

$$q_0 = \gamma b h \quad S = \frac{bt^2}{6}$$

MAXIMUM BENDING MOMENT



$$R_A = \frac{q_0 L}{6}$$

$$M = R_A x - \frac{q_0 x^3}{6L}$$

$$= \frac{q_0 L x}{6} - \frac{q_0 x^3}{6L}$$

$$\frac{dM}{dx} = \frac{q_0 L}{6} - \frac{q_0 x^2}{2L} = 0 \quad x = \frac{L}{\sqrt{3}}$$

Substitute $x = L/\sqrt{3}$ into the equation for M:

$$M_{max} = \frac{q_0 L}{6}\left(\frac{L}{\sqrt{3}}\right) - \frac{q_0}{6L}\left(\frac{L^3}{3\sqrt{3}}\right) = \frac{q_0 L^2}{9\sqrt{3}}$$

For the vertical wood beam: $L = h$; $M_{max} = \dfrac{q_0 h^2}{9\sqrt{3}}$

Maximum bending stress

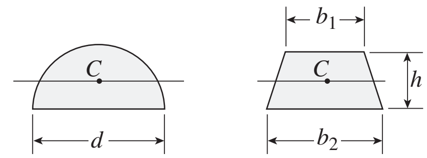
$$\sigma_{max} = \frac{M_{max}}{S} = \frac{2 q_0 h^2}{3\sqrt{3}\, b t^2} = \frac{2\gamma h^3}{3\sqrt{3}\, t^2}$$

SUBSTITUTE NUMERICAL VALUES:

$\sigma_{max} = 2.10$ MPa ←

NOTE: For $b = 1.0$ m, we obtain $q_0 = 19{,}620$ N/m, $S = 0.0024$ m^3, $M_{max} = 5{,}034.5$ N · m, and $\sigma_{max} = M_{max}/S = 2.10$ MPa

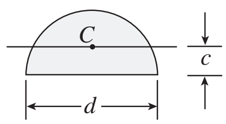
Problem 5.5-13 Determine the maximum tensile stress σ_t (due to pure bending by positive bending moments M) for beams having cross sections as follows (see figure): (a) a semicircle of diameter d, and (b) an isosceles trapezoid with bases $b_1 = b$ and $b_2 = 4b/3$, and altitude h.



(a) (b)

Solution 5.5-13 **Maximum tensile stress**

(a) SEMICIRCLE



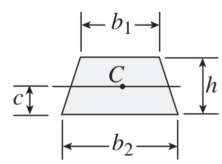
From Appendix D, Case 10:

$$I_C = \frac{(9\pi^2 - 64)r^4}{72\pi} = \frac{(9\pi^2 - 64)d^4}{1152\pi}$$

$$c = \frac{4r}{3\pi} = \frac{2d}{3\pi}$$

$$\sigma_t = \frac{Mc}{I_C} = \frac{768M}{(9\pi^2 - 64)d^3} = 30.93\,\frac{M}{d^3} \quad \longleftarrow$$

(b) TRAPEZOID



$$b_1 = b \qquad b_2 = \frac{4b}{3}$$

From Appendix D, Case 8:

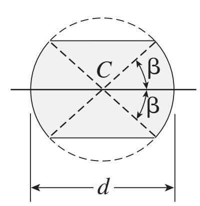
$$I_C = \frac{h^3(b_1^2 + 4b_1b_2 + b_2^2)}{36(b_1 + b_2)}$$

$$= \frac{73bh^3}{756}$$

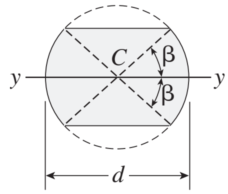
$$c = \frac{h(2b_1 + b_2)}{3(b_1 + b_2)} = \frac{10h}{21}$$

$$\sigma_t = \frac{Mc}{I_C} = \frac{360M}{73bh^2} \quad \longleftarrow$$

Problem 5.5-14 Determine the maximum bending stress σ_{max} (due to pure bending by a moment M) for a beam having a cross section in the form of a circular core (see figure). The circle has diameter d and the angle $\beta = 60°$. (*Hint:* Use the formulas given in Appendix D, Cases 9 and 15.)



Solution 5.5-14 Circular core



From Appendix D, Cases 9 and 15:

$$I_y = \frac{\pi r^4}{4} - \frac{r^4}{2}\left(\alpha - \frac{ab}{r^2} + \frac{2ab^3}{r^4}\right)$$

$$r = \frac{d}{2} \quad \alpha = \frac{\pi}{2} - \beta$$

$\beta =$ radians $\quad \alpha =$ radians $\quad a = r\sin\beta \quad b = r\cos\beta$

$$I_y = \frac{\pi d^4}{64} - \frac{d^4}{32}\left(\frac{\pi}{2} - \beta - \sin\beta\cos\beta + 2\sin\beta\cos^3\beta\right)$$

$$= \frac{\pi d^4}{64} - \frac{d^4}{32}\left(\frac{\pi}{2} - \beta - (\sin\beta\cos\beta)(1 - 2\cos^2\beta)\right)$$

$$= \frac{\pi d^4}{64} - \frac{d^4}{32}\left(\frac{\pi}{2} - \beta - \left(\frac{1}{2}\sin 2\beta\right)(-\cos 2\beta)\right)$$

$$= \frac{\pi d^4}{64} - \frac{d^4}{32}\left(\frac{\pi}{2} - \beta + \frac{1}{4}\sin 4\beta\right)$$

$$= \frac{d^4}{128}(4\beta - \sin 4\beta)$$

Maximum bending stress

$$\sigma_{max} = \frac{Mc}{I_y} \quad c = r\sin\beta = \frac{d}{2}\sin\beta$$

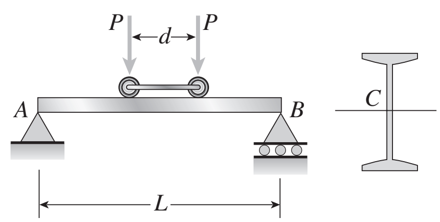
$$\sigma_{max} = \frac{64M\sin\beta}{d^3(4\beta - \sin 4\beta)} \quad \longleftarrow$$

For $\beta = 60° = \pi/3$ *rad:*

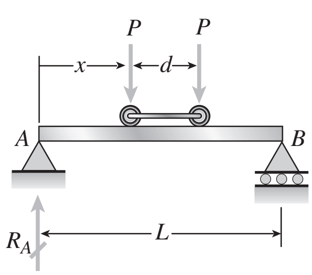
$$\sigma_{max} = \frac{576M}{(8\pi\sqrt{3} + 9)d^3} = 10.96\frac{M}{d^3} \quad \longleftarrow$$

Problem 5.5-15 A simple beam AB of span length $L = 24$ ft is subjected to two wheel loads acting at distance $d = 5$ ft apart (see figure). Each wheel transmits a load $P = 3.0$ k, and the carriage may occupy any position on the beam.

Determine the maximum bending stress σ_{max} due to the wheel loads if the beam is an I-beam having section modulus $S = 16.2$ in.^{3}



Solution 5.5-15 Wheel loads on a beam



$L = 24$ ft $= 288$ in.
$d = 5$ ft $= 60$ in.
$P = 3$ k
$S = 16.2$ in.^{3}

Maximum bending moment

$$R_A = \frac{P}{L}(L - x) + \frac{P}{L}(L - x - d) = \frac{P}{L}(2L - d - 2x)$$

$$M = R_A x = \frac{P}{L}(2Lx - dx - 2x^2)$$

$$\frac{dM}{dx} = \frac{P}{L}(2L - d - 4x) = 0 \quad x = \frac{L}{2} - \frac{d}{4}$$

Substitute x into the equation for M:

$$M_{max} = \frac{P}{2L}\left(L - \frac{d}{2}\right)^2$$

Maximum bending stress

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{P}{2LS}\left(L - \frac{d}{2}\right)^2 \quad \longleftarrow$$

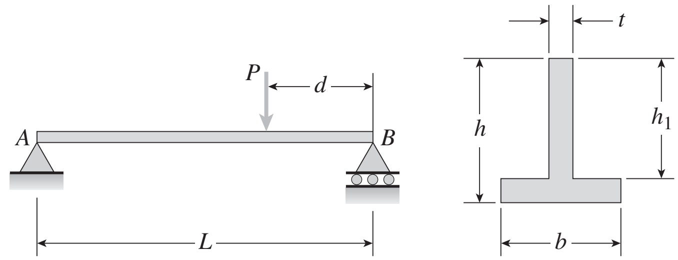
Substitute numerical values:

$$\sigma_{max} = \frac{3\text{k}}{2(288\text{ in.})(16.2\text{ in.}^3)}(288\text{ in.} - 30\text{ in.})^2$$

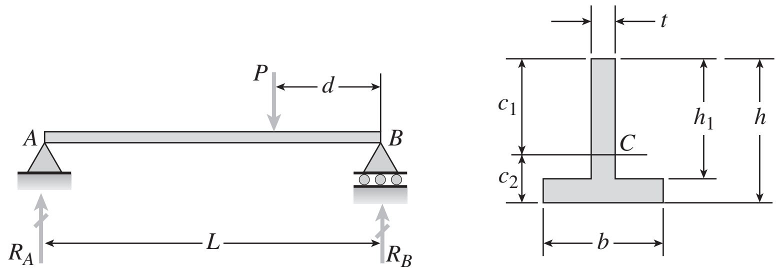
$$= 21.4\text{ ksi} \quad \longleftarrow$$

Problem 5.5-16 Determine the maximum tensile stress σ_t and maximum compressive stress σ_c due to the load P acting on the simple beam AB (see figure).

Data are as follows: $P = 5.4$ kN, $L = 3.0$ m, $d = 1.2$ m, $b = 75$ mm, $t = 25$ mm, $h = 100$ mm, and $h_1 = 75$ mm.



Solution 5.5-16 **Simple beam of T-section**



$P = 5.4$ kN $L = 3.0$ m

$b = 75$ mm $t = 25$ mm

$d = 1.2$ m $h = 100$ mm $h_1 = 75$ mm

PROPERTIES OF THE CROSS SECTION

$A = 3750 \text{ mm}^2$

$c_1 = 62.5$ mm $c_2 = 37.5$ mm

$I_C = 3.3203 \times 10^6 \text{ mm}^4$

REACTIONS OF THE BEAM

$R_A = 2.16$ kN $R_B = 3.24$ kN

MAXIMUM BENDING MOMENT

$$M_{max} = R_A(L - d) = R_B(d) = 3888 \text{ N} \cdot \text{m}$$

MAXIMUM TENSILE STRESS

$$\sigma_t = \frac{M_{max} c_2}{I_C} = \frac{(3888 \text{ N} \cdot \text{m})(0.0375 \text{ m})}{3.3203 \times 10^6 \text{ mm}^4}$$

$$= 43.9 \text{ MPa} \quad \longleftarrow$$

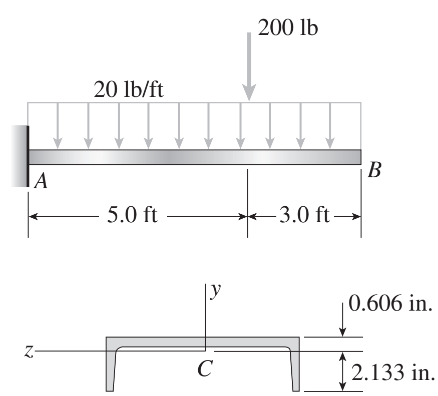
MAXIMUM COMPRESSIVE STRESS

$$\sigma_c = \frac{M_{max} c_1}{I_C} = \frac{(3888 \text{ N} \cdot \text{m})(0.0625 \text{ m})}{3.3203 \times 10^6 \text{ mm}^4}$$

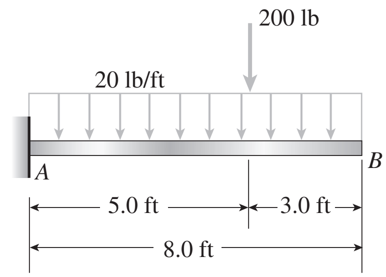
$$= 73.2 \text{ MPa} \quad \longleftarrow$$

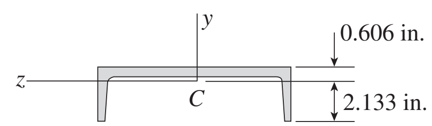
Problem 5.5-17 A cantilever beam AB, loaded by a uniform load and a concentrated load (see figure), is constructed of a channel section.

Find the maximum tensile stress σ_t and maximum compressive stress σ_c if the cross section has the dimensions indicated and the moment of inertia about the z axis (the neutral axis) is $I = 2.81 \text{ in.}^4$ (*Note:* The uniform load represents the weight of the beam.)



Solution 5.5-17 **Cantilever beam (channel section)**





$I = 2.81 \text{ in.}^4 \quad c_1 = 0.606 \text{ in.} \quad c_2 = 2.133 \text{ in.}$

$$M_{\max} = (200 \text{ lb})(5.0 \text{ ft}) + (20 \text{ lb/ft})(8.0 \text{ ft})\left(\frac{8.0 \text{ ft}}{2}\right)$$
$$= 1000 \text{ lb-ft} + 640 \text{ lb-ft} = 1640 \text{ lb-ft}$$
$$= 19{,}680 \text{ lb-in.}$$

Maximum tensile stress

$$\sigma_t = \frac{Mc_1}{I} = \frac{(19{,}680 \text{ lb-in.})(0.606 \text{ in.})}{2.81 \text{ in.}^4}$$
$$= 4{,}240 \text{ psi} \longleftarrow$$

Maximum compressive stress

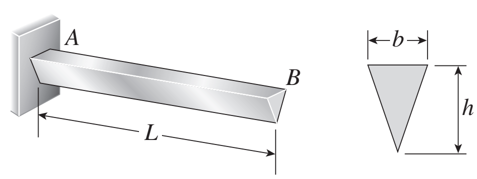
$$\sigma_c = \frac{Mc_2}{I} = \frac{(19{,}680 \text{ lb-in.})(2.133 \text{ in.})}{2.81 \text{ in.}^4}$$
$$= 14{,}940 \text{ psi} \longleftarrow$$

Problem 5.5-18 A cantilever beam AB of triangular cross section has length $L = 0.8$ m, width $b = 80$ mm, and height $h = 120$ mm (see figure). The beam is made of brass weighing 85 kN/m^3.

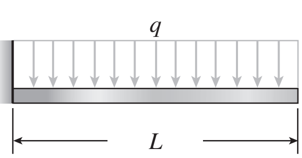
(a) Determine the maximum tensile stress σ_t and maximum compressive stress σ_c due to the beam's own weight.

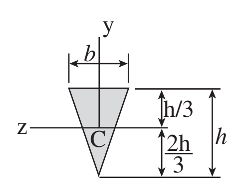
(b) If the width b is doubled, what happens to the stresses?

(c) If the height h is doubled, what happens to the stresses?



Solution 5.5-18 **Triangular beam**





$L = 0.8 \text{ m} \quad b = 80 \text{ mm} \quad h = 120 \text{ mm}$
$\gamma = 85 \text{ kN/m}^3$

(a) Maximum stresses

$$q = \gamma A = \gamma\left(\frac{bh}{2}\right) \quad M_{\max} = \frac{qL^2}{2} = \frac{\gamma b h L^2}{4}$$

$$I_z = I_C = \frac{bh^3}{36} \quad c_1 = \frac{h}{3} \quad c_2 = \frac{2h}{3}$$

Tensile stress: $\sigma_t = \dfrac{Mc_1}{I_z} = \dfrac{3\gamma L^2}{h}$

Compressive stress: $\sigma_c = 2\sigma_t$

Substitute numerical values: $\sigma_t = 1.36$ MPa $\longleftarrow$

$\sigma_c = 2.72$ MPa $\longleftarrow$

(b) Width b is doubled

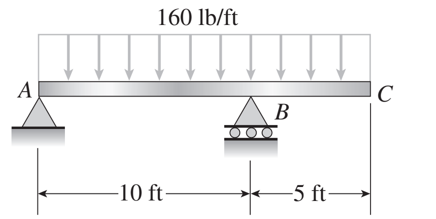
No change in stresses. $\longleftarrow$

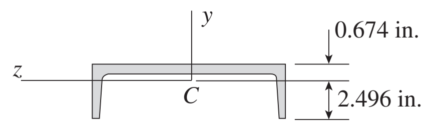
(c) Height h is doubled

Stresses are reduced by half. $\longleftarrow$

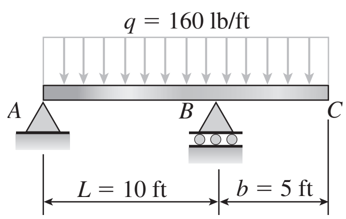
Problem 5.5-19 A beam *ABC* with an overhang from *B* to *C* supports a uniform load of 160 lb/ft throughout its length (see figure). The beam is a channel section with dimensions as shown in the figure. The moment of inertia about the *z* axis (the neutral axis) equals 5.14 in.^{4}

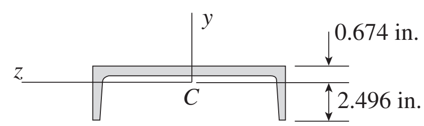
Calculate the maximum tensile stress σ_t and maximum compressive stress σ_c due to the uniform load.

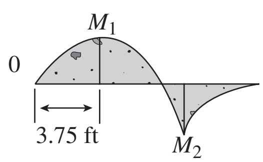




Solution 5.5-19 **Beam with an overhang**







$I_z = 5.14 \text{ in.}^4$

$c_1 = 0.674 \text{ in.} \quad c_2 = 2.496 \text{ in.}$

$R_A = 600 \text{ lb} \quad R_B = 1800 \text{ lb}$

$M_1 = 1125 \text{ lb-ft} = 13{,}500 \text{ lb-in.}$

$M_2 = 2000 \text{ lb-ft} = 24{,}000 \text{ lb-in.}$

At cross section of maximum positive bending moment

$$\sigma_t = \frac{M_1 c_2}{I_z} = \frac{(13{,}500 \text{ lb-in.})(2.496 \text{ in.})}{5.14 \text{ in.}^4} = 6{,}560 \text{ psi}$$

$$\sigma_c = \frac{M_1 c_1}{I_z} = \frac{(13{,}500 \text{ lb-in.})(0.674 \text{ in.})}{5.14 \text{ in.}^4} = 1{,}770 \text{ psi}$$

At cross section of maximum negative bending moment

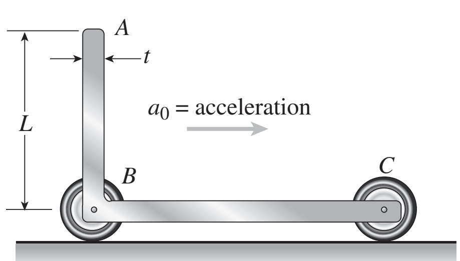
$$\sigma_t = \frac{M_2 c_1}{I_z} = \frac{(24{,}000 \text{ lb-in.})(0.674 \text{ in.})}{5.14 \text{ in.}^4} = 3{,}150 \text{ psi}$$

$$\sigma_c = \frac{M_2 c_2}{I_z} = \frac{(24{,}000 \text{ lb-in.})(2.496 \text{ in.})}{5.14 \text{ in.}^4} = 11{,}650 \text{ psi}$$

Maximum stresses

$\sigma_t = 6{,}560 \text{ psi} \quad \sigma_c = 11{,}650 \text{ psi}$ ←

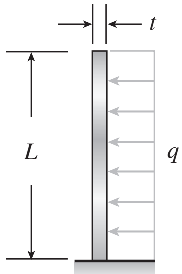
Problem 5.5-20 A frame *ABC* travels horizontally with an acceleration a_0 (see figure). Obtain a formula for the maximum stress σ_{max} in the vertical arm *AB*, which has length *L*, thickness *t*, and mass density *ρ*.



Solution 5.5-20 Accelerating frame

L = length of vertical arm
t = thickness of vertical arm
ρ = mass density
a_0 = acceleration
Let b = width of arm perpendicular to the plane of the figure
Let q = inertia force per unit distance along vertical arm

VERTICAL ARM



$$q = \rho b t a_0 \qquad M_{max} = \frac{qL^2}{2} = \frac{\rho b t a_0 L^2}{2}$$

$$S = \frac{bt^2}{6} \qquad \sigma_{max} = \frac{M_{max}}{S} = \frac{3\rho L^2 a_0}{t} \quad \longleftarrow$$

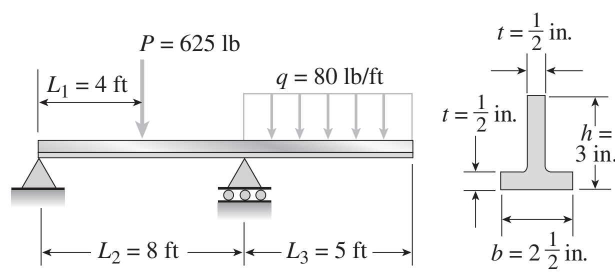
TYPICAL UNITS FOR USE IN THE PRECEDING EQUATION

SI UNITS: $\rho = \text{kg/m}^3 = \text{N} \cdot \text{s}^2/\text{m}^4$
L = meters (m)
$a_0 = \text{m/s}^2$
t = meters (m)
$\sigma_{max} = \text{N/m}^2$ (pascals)

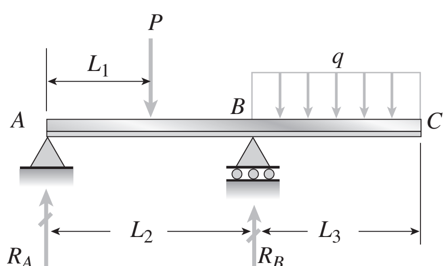
USCS UNITS: $\rho = \text{slug/ft}^3 = \text{lb-s}^2/\text{ft}^4$
L = ft $\quad a_0 = \text{ft/s}^2 \quad t$ = ft
$\sigma_{max} = \text{lb/ft}^2$ (Divide by 144 to obtain psi)

Problem 5.5-21 A beam of T-section is supported and loaded as shown in the figure. The cross section has width b =2 1/2 in., height h = 3 in., and thickness t = 1/2 in.

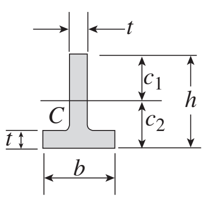
Determine the maximum tensile and compressive stresses in the beam.



Solution 5.5-21 Beam of T-section



L_1 = 4 ft = 48 in. $\quad L_2$ = 8 ft = 96 in. $\quad L_3$ = 5 ft = 60 in.
P = 625 lb $\quad q$ = 80 lb/ft = 6.6667 lb/in.



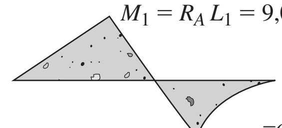
PROPERTIES OF THE CROSS SECTION

b = 2.5 in. $\quad h$ = 3.0 in. $\quad t$ = 0.5 in.
$A = bt + (h - t)t = 2.50 \text{ in.}^2$
c_1 = 2.0 in. $\quad c_2$ = 1.0 in.
$$I_C = \frac{25}{12} \text{ in.}^4 = 2.0833 \text{ in.}^4$$

REACTIONS

R_A = 187.5 lb (upward)
R_B = 837.5 lb (upward)

BENDING-MOMENT DIAGRAM



$M_1 = R_A L_1$ = 9,000 lb – in.

$$M_2 = \frac{-qL_3^2}{2} = -12{,}000 \text{ lb – in.}$$

AT CROSS SECTION OF MAXIMUM POSITIVE BENDING MOMENT

$$\sigma_t = \frac{M_1 c_2}{I_C} = 4{,}320 \text{ psi} \qquad \sigma_c = \frac{M_1 c_1}{I_C} = 8{,}640 \text{ psi}$$

AT CROSS SECTION OF MAXIMUM NEGATIVE BENDING MOMENT

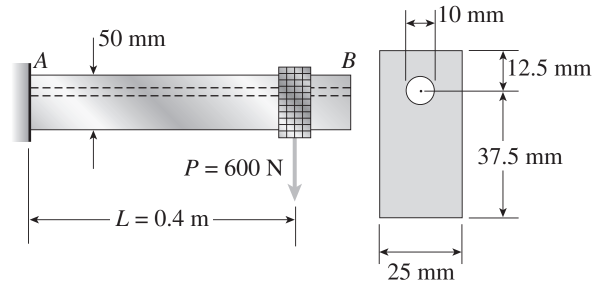
$$\sigma_t = \frac{M_2 c_1}{I_C} = 11{,}520 \text{ psi} \qquad \sigma_c = \frac{M_2 c_2}{I_C} = 5{,}760 \text{ psi}$$

MAXIMUM STRESSES

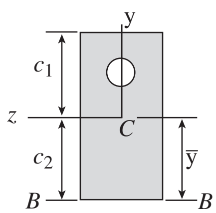
σ_t = 11,520 psi $\quad \sigma_c$ = 8,640 psi $\longleftarrow$

Problem 5.5-22 A cantilever beam AB with a rectangular cross section has a longitudinal hole drilled throughout its length (see figure). The beam supports a load $P = 600$ N. The cross section is 25 mm wide and 50 mm high, and the hole has a diameter of 10 mm.

Find the bending stresses at the top of the beam, at the top of the hole, and at the bottom of the beam.



Solution 5.5-22 **Rectangular beam with a hole**



Maximum bending moment

$M = PL = (600\text{ N})(0.4\text{ m}) = 240\text{ N}\cdot\text{m}$

Properties of the cross section

A_1 = area of rectangle

$= (25\text{ mm})(50\text{ mm}) = 1250\text{ mm}^2$

A_2 = area of hole

$= \dfrac{\pi}{4}(10\text{ mm})^2 = 78.54\text{ mm}^2$

A = area of cross section

$= A_1 - A_2 = 1171.5\text{ mm}^2$

Using line B-B as reference axis:

$\sum A_i y_i = A_1(25\text{ mm}) - A_2(37.5\text{ mm}) = 28{,}305\text{ mm}^3$

$$\bar{y} = \frac{\sum A_i y_i}{A} = \frac{28{,}305\text{ mm}^3}{1171.5\text{ mm}^2} = 24.162\text{ mm}$$

Distances to the centroid C:

$c_2 = \bar{y} = 24.162\text{ mm}$

$c_1 = 50\text{ mm} - c_2 = 25.838\text{ mm}$

Moment of inertia about the neutral axis (the z axis)

All dimensions in millimeters.

Rectangle:

$$I_z = I_c + Ad^2$$
$$= \frac{1}{12}(25)(50)^3 + (25)(50)(25 - 24.162)^2$$
$$= 260{,}420 + 878 = 261{,}300\text{ mm}^4$$

Hole:

$$I_z = I_c + Ad^2 = \frac{\pi}{64}(10)^4 + (78.54)(37.5 - 24.162)^2$$
$$= 490.87 + 13{,}972 = 14{,}460\text{ mm}^4$$

Cross-section:

$I = 261{,}300 - 14{,}460 = 246{,}800\text{ mm}^4$

Stress at the top of the beam

$$\sigma_1 = \frac{Mc_1}{I} = \frac{(240\text{ N}\cdot\text{m})(25.838\text{ mm})}{246{,}800\text{ mm}^4}$$
$$= 25.1\text{ MPa} \longleftarrow$$
(tension)

Stress at the top of the hole

$$\sigma_2 = \frac{My}{I} \qquad y = c_1 - 7.5\text{ mm} = 18.338\text{ mm}$$

$$\sigma_2 = \frac{(240\text{ N}\cdot\text{m})(18.338\text{ mm})}{246{,}800\text{ mm}^4} = 17.8\text{ MPa} \longleftarrow$$

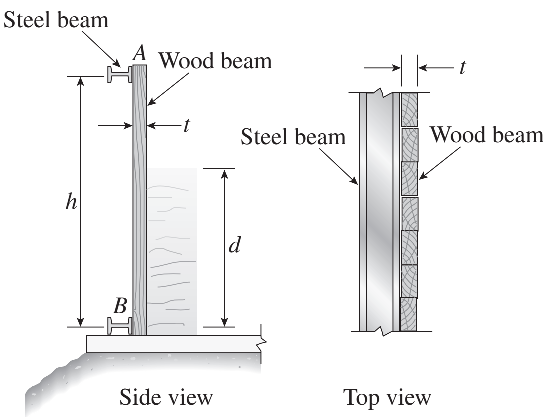
(tension)

Stress at the bottom of the beam

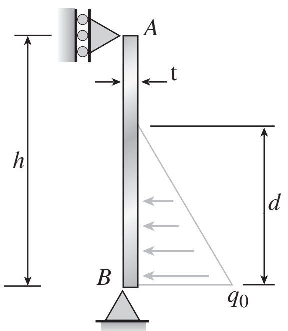
$$\sigma_3 = -\frac{Mc_2}{I} = -\frac{(240\text{ N}\cdot\text{m})(24.162\text{ mm})}{246{,}800\text{ mm}^4}$$
$$= -23.5\text{ MPa} \longleftarrow$$
(compression)

Problem 5.5-23 A small dam of height $h = 6$ ft is constructed of vertical wood beams AB, as shown in the figure. The wood beams, which have thickness $t = 2.5$ in., are simply supported by horizontal steel beams at A and B.

Construct a graph showing the maximum bending stress σ_{max} in the wood beams versus the depth d of the water above the lower support at B. Plot the stress σ_{max} (psi) as the ordinate and the depth d (ft) as the abscissa. (*Note:* The weight density γ of water equals 62.4 lb/ft^3.)

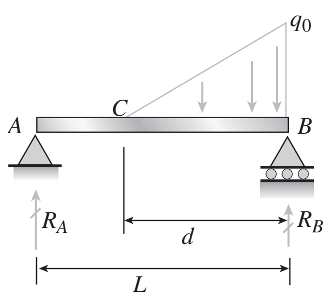


Solution 5.5-23 **Vertical wood beam in a dam**



$h = 6$ ft
$t = 2.5$ in.
$\gamma = 62.4$ lb/ft^3
Let b = width of beam (perpendicular to the figure)
Let q_0 = intensity of load at depth d
$q_0 = \gamma bd$

ANALYSIS OF BEAM



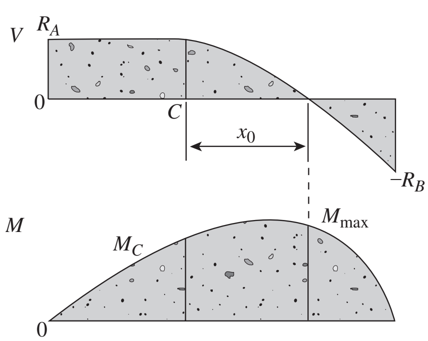
$L = h = 6$ ft

$$R_A = \frac{q_0 d^2}{6L}$$

$$R_B = \frac{q_0 d}{6}\left(3 - \frac{d}{L}\right)$$

$$x_0 = d\sqrt{\frac{d}{3L}}$$

$$M_C = R_A(L - d) = \frac{q_0 d^2}{6}\left(1 - \frac{d}{L}\right)$$



$$M_{max} = \frac{q_0 d^2}{6}\left(1 - \frac{d}{L} + \frac{2d}{3L}\sqrt{\frac{d}{3L}}\right)$$

MAXIMUM BENDING STRESS

Section modulus: $S = \frac{1}{6}bt^2$

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{6}{bt^2}\left[\frac{q_0 d^2}{6}\left(1 - \frac{d}{L} + \frac{2d}{3L}\sqrt{\frac{d}{3L}}\right)\right]$$

$q_0 = \gamma bd$

$$\sigma_{max} = \frac{\gamma d^3}{t^2}\left(1 - \frac{d}{L} + \frac{2d}{3L}\sqrt{\frac{d}{3L}}\right) \quad \longleftarrow$$

SUBSTITUTE NUMERICAL VALUES:

d = depth of water (ft) (Max. $d = h = 6$ ft)
$L = h = 6$ ft $\quad \gamma = 62.4$ lb/ft^3 $\quad t = 2.5$ in.
σ_{max} = psi

$$\sigma_{max} = \frac{(62.4)d^3}{(2.5)^2}\left(1 - \frac{d}{6} + \frac{d}{9}\sqrt{\frac{d}{18}}\right)$$

$$= 0.1849d^3(54 - 9d + d\sqrt{2d}) \quad \longleftarrow$$

d (ft)	σ_{max} (psi)
0	0
1	9
2	59
3	171
4	347
5	573
6	830